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Grand Unification of Fermion Masses *

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Abstract

After a brief review of the flavour problem we present a new predictive framework based on SUSY $SO(10)$ theory, where the first family plays a role of the mass unification point. The inter-family hierarchy is first generated in a sector of superheavy fermions and then transfered in an inverse way to ordinary quarks and leptons by means of the universal seesaw mechanism. The obtained mass matrices are simply parametrized by two small coefficients which can be given by the ratio of the GUT and superstring compactification scales. The model allows a natural (without fine tuning) doublet-triplet splitting. It has a strong predictive power, though no special texture is utilized in contrast to the known predictive frameworks. Namely, the model implies that $m_b = 4-5$ GeV, $m_s = 100-150$ MeV, $m_u/m_d = 0.5-0.7$ and $\tan\beta < 1.1$. The Top quark is naturally in the 100 GeV range, but not too heavy: $m_t < 150$ GeV. All CKM mixing angles are in correct range. The Higgsino mediated $d = 5$ operators for the proton decay are naturally suppressed.

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1. Introduction to Family Puzzles

The Standard Model (SM) is internally consistent from the field theoretical view, and has been extremely successfull in describing various experimental data accumulated over the past several years. This suggests that at presently available energies the SM is literally correct in all its sectors. It is widely believed, however, that there should be a more fundamental theory valid at some higher energies. The most important issues that motivate such a belief include the unification of gauge couplings, problem of gauge hierarchies and problem of fermion flavours (or families).

In SM all the observed fermions are accomodated in a consistent way. Three families share the same quantum numbers under the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry. Each family is considered as an anomaly free set of initially massless chiral fermions. They become massive due to the same Higgs field that gives masses to $W^{\pm}$ and Z bosons. An important feature of the minimal SM is that the flavour changing neutral currents (FCNC) are naturally suppressed in both gauge and Higgs boson exchanges [1]. However, the pattern of fermion masses and mixing remains undetermined due to arbitrariness of the Yukawa couplings. A hypothetical fundamental theory should allow to calculate these couplings, or at least somehow constrain them. When thinking of such a theory, one should bear in mind that the flavour problem has different aspects, questioning the origin of (i) family replication (ii) mass and mixing pattern of charged fermions (iii) CP violation in weak interactions (iv) CP conservation in strong interactions (v) tiny neutrino masses.

There is an almost holy trust that all the fundamental problems, and among them the problem of fermion flavours, will find a final solution within the Superstring Theory = Theory of Everything. In principle, it should allow us to *calculate* all Yukawa couplings. Unfortunatelly, it is our lack of understanding how the superstring can be linked in unambiguous way to lower energy physics. Many theorists try to attack the problem in whole, or its certain aspects, in the context of particular (among many billions) superstring inspired models. However, the problem remains far away from being solved and all what we know at present from superstring can be updated in few important but rather general recommendations.

Nevertheless, one may rely that many aspects of the flavour problem can be understood by means of more familiar symmetry properties. A relevant theory could take place at some intermediate energies between the electroweak and Planck scales. Nowadays the concepts of grand unification and supersymetry are the most promising ideas towards the physics beyond the SM, providing a sound basis for understanding the issues of coupling unification and stability of gauge hierarchy. In particular, the famous coupling crossing phenomenon in the Minimal Supersymmetric Standard Model (MSSM) points to the GUT scale $M_G \simeq 10^{16}$ GeV [2]. Applied to the flavour problem, these ideas should be complemented by introducing some inter-family symmetry $\mathcal{H}$ that could constrain the structure of fermion mass matrices. It is natural to expect that $\mathcal{H}$ is also broken at the GUT scale M_G. Such a $SUSY \otimes GUT \otimes \mathcal{H}$ theory can be regarded as a Grand Unification of fermion

masses. Here I shall review some aspects of the flavour physics, and suggest a new possibility that could shed some more light towards the search of such a theory.

The most difficult question is originated by the fact of family replication itself. The measurement of the Z-boson decay width supports the idea that there are just three observed families; at least, we are sure that there are no more standard-like families with light neutrinos. There seems to be no simple answer to the question "why three families?", or "why *only* three families?". Apparently it is in the competence of superstring, and I have nothing to add here. In what follows, I shall simply accept that there are three families and pursue the understanding of the issues (ii)-(v). Let us first outline these issues from the SM point of view.

The mass spectrum of the quarks and charged leptons is spread over five orders of magnitude, from MeVs to 100 GeVs [3]. In order to understand its shape it is necessary to compare the fermion running masses at some scale $\mu \sim M_G$, where the relevant new physics could take the place.[1)] In doing so, we see that the *horizontal* hierarchy of quark masses exhibits the approximate scaling low (see Fig. 1)

$$t : c : u \sim 1 : \varepsilon_u : \varepsilon_u^2 \,, \qquad b : s : d \sim 1 : \varepsilon_d : \varepsilon_d^2 \tag{1}$$

where $\varepsilon_u^{-1} = 200 - 300$ and $\varepsilon_d^{-1} = 20 - 30$. As for the charged leptons, they have a mixed behaviour:

$$\tau : \mu : e \sim 1 : \varepsilon_d : \varepsilon_u \varepsilon_d \tag{2}$$

Fig. 1. Logaritmic plot of fermion running masses at the GUT scale versus family number. Points corresponding to the fermions with the same electric charge are joined. The value m_t=130 GeV has been assumed.

[1)] In what follows, with an obvious notation we indicate by $u, d, ...$ the fermion running masses at the GUT scale, and by $m_u, m_d, ...$ their physical masses. For the light quarks (u,d,s) the latter traditionally are taken as running masses at $\mu = 1$ GeV [4].

One can also observe that the *vertical* mass splitting is small within the first family of quarks and is quickly growing with the family number:

$$\frac{u}{d} \sim \frac{1}{2}\,, \qquad \frac{c}{s} \sim 8\,, \qquad \frac{t}{b} \sim 60\,, \tag{3}$$

whereas the splitting between the charged leptons and down quarks (at large μ) remains considerably smaller (see Fig. 1). Moreover, we observe that the third family is almost unsplit, $b \approx \tau$, whereas the first two families are split but $ds \approx e\mu$.

One can also exploit experimental information on the quark mixing. The weak transitions dominantly occur inside the families, and are suppressed between different families by the small Cabibbo-Kobayashi-Maskawa (CKM) angles [5]:

$$s_{12} \sim \varepsilon_d^{1/2}\,, \qquad s_{23} \sim \varepsilon_d\,, \qquad s_{13} \sim \varepsilon_d^2\,, \tag{4}$$

This shows that the quark mass spectrum and weak mixing pattern are strongly correlated. Moreover, there are intriguing relations between masses and mixing angles, such as the well-known formula $s_{12} = \sqrt{d/s}$ for the Cabibbo angle.

All of the observed CP-violating phenomena [6] can be successfully described in the frames of the SM due to sufficiently large ($\delta \sim 1$) CP-phase in the CKM matrix.[2)] This means that the fermion mass matrices are complex. The strong CP problem is closely related to this issue: the net phase of the complex mass matrices should effectively contribute to the P and CP violating Θ-term, whereas an absence of the neutron dipole electric moment puts a strong bound $\Theta < 10^{-9}$ [8].

As for neutrino masses and mixing, only some experimental upper bounds are in our disposal [3]. If SM is true all the way up to Planckian energies, the neutrinos would stay massless, or could get some tiny masses ($\sim 10^{-5}$ eV) due to non-perturbative quantum gravitational (or superstring) effects [9]. However, some rather astrophysical data, as are the solar and atmospheric neutrino deficites or the "after COBE" evidence for some *hot* fraction of the cosmological dark matter, may hint in favour of rather heavier and substantially mixed neutrinos.

As noted above, in the SM the fermion mass and mixing problem can be phrased as a problem of the Yukawa coupling matrices: there is no explanation, what is the origin of such a strong hierarchy of their eigenvalues, why they are alligned approximately for the up and down quarks, what is the origin of their complex structure, why the Θ-term is vanishing in spite of this complex structure etc. On the other hand, we believe that the SM (or rather MSSM [10]) is literally correct at lower energies. This provides a "boundary" condition for any hypothetical theory of the flavour: in the low energy limit it should reduce to the minimal SM (or MSSM) in *all* sectors, i.e. all the possible extra degrees of freedom must decouple. Since the decoupling is expected to occur at superhigh ($\sim M_G$) energies, there is practically

[2)]It is tempting to mention that even an outstanding issue of the cosmological baryon asymmetry can be accounted entirely (with correct sign and magnitude) within the minimal SM, for the reasonable values of the CKM angles and CP-phase [7].

no hope to observe experimentally any direct dynamical effect of such a theory of the flavour. It could manifest itself only in the sense of the flavour statics, through certain constraints on the Yukawa sector of the resulting SM, or, in other words, through the testable predictions for the fermion masses and CKM parameters. In particular, it is tempting to think that the certain structure of the mass matrices is responsible for several mass relations between fermions and the CKM angles can be expressed as a functions of these masses. It is also suggestive to think that these functions have the following "analytic" properties [11]:

Decoupling Hypothesis. The mixing angles of the first quark family with others (s_{12}, s_{13}) vanish in the limit $u, d \to 0$. At the next step, when $c, s \to 0$, s_{23} also vanishes.

Scaling Hypothesis. In the limit when the masses of the up and down quarks are proportional to each other: $u : c : t = d : s : b$, all mixing angles (s_{12}, s_{13} and s_{23}) are vanishing.

Therefore, our purpose is to find a self-consistent, complete and elegant enough example of such a theory that could provide, besides solving other fundamental problems, a natural explanation to the fermion mass and mixing pattern.

"Models? No problem. We have many models."
R.Mohapatra

2. Mass Matrix Models

Even the simplest extensions of the SM symmetry $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ can provide some interesting hints for the understanding of fermion mass and mixing pattern. For example, already the minimal grand unified theory $SU(5)$ [12] points to the possible origin of $b \approx \tau$, specifying also that this relation should take place at the GUT scale M_G. The SUSY $SU(5)$ theory, which is more appealing from the viewpoint of internal consistency, leads to better quantitative agreement for the $b - \tau$ unification, as well as for the gauge coupling unification [13]. Unfortunately, the analogous relations $s = \mu$ and $d = e$ are simply wrong, and one is forced to invoke some extra sources (e.g. Planck scale induced higher dimensional operators [14]), in order to split these masses from each other. also, there is no hint neither for the origin of the fermion mass hierarchy nor for the smallnes of the quark mixing angles: in the $SU(5)$ theory, as well as in the SM, the Yukawa couplings for the up and down type fermions are different and there is no reason for their allignment.

Another minimal extension of the SM, so called $L-R$ model $G_{LR} = SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ provides an "isotopic" symmetry interchanging the up and down fermions, so that their mass matrices are somehow alligned. Therefore, the smallness of quark mixing angles can be naturally linked to the horizontal hierarchy of he quark masses, though the origin of the hierarchy itself is beyond the scope of this model. Also, an additional discrete $L \leftrightarrow R$ symmetry, essentially P-parity, can be imposed naturally for the constraining of fermion mass matrices.

On the other hand, the G_{LR} model does not imply any relations between the quark and lepton masses, as the $SU(5)$ does. Its further extension to the Pati-Salam theory $G_{PS} = SU(4) \otimes SU(2)_L \otimes SU(2)_R$ unifies the leptons with quarks as a fourth colour, and thereby provides the possibility for the $b - \tau$ unification. However, it does not determine the scale of this unification.

The ends are closed in the $SO(10)$ GUT (or rather SUSY GUT), which accomodates each family of quarks and leptons of both chiralities within the spinorial representation 16. $SO(10)$ is a logical final towards both chains of the SM extensions: $G_{SM} \to SU(5) \to SO(10)$, or $G_{SM} \to G_{LR} \to G_{PS} \to SO(10)$. Therefore, the $SO(10)$ naturally contains all types of the simplest fermionic symmetries: the isotopic and quark-lepton symmetries as well as automatic P-parity.

All these appear to be necessary but not sufficient tools for the fermion mass model building: also some inter-family (horizontal) symmetries should be invoked in order to constrain the fermion mass matrices at the needed degree. In the literature there are two main directions in the flavour physics, which generally do not have strong intersection. These are: (i) mass matrix ansatzes, and (ii) radiative mass generation.

The general aim of the first direction [15, 16, 17] is to provide *predictivity*, i.e. certain relations between fermion masses and the CKM angles, by constraining the mass matrix form and by reducing the number of its parameters. This can be motivated by some family symmetries (or, in some cases, even are not motivated). In general, this implies a study of the so called "zero textures" - matrices with the certain number of zero elements. In order to reduce a number of arbitrary parameters, together with horizontal symmetry one should utilize the above mentioned GUT ingredients as are the isotopic and quark-lepton symmetry and P-parity. One of the most interesting mass matrix ansatzes is given by Fritzsch texture [16]

$$\hat{m}_f = \begin{pmatrix} 0 & A_f & 0 \\ A'_f & 0 & B_f \\ 0 & B'_f & C_f \end{pmatrix}, \qquad f = u, d, e \tag{5}$$

which can be obtained at the price of some horizontal symmetry. This structure implies that the fermion mass generation starts from the heavist 3^{rd} family (C is a largest entry in eq. (5)) and proceeds to lighter families through the mixing terms. In the context of the $L-R$ symmetric model this matrices can be Hermitian due to P-parity. Thereby, neglecting the phase factors, the total number of the parameters for 3 mass matrices ($f = u, d, e$) is reduced to a number 9, i.e. just to the number of quarks and leptons. This allows to express the quark mixing angles in terms of their mass ratios: in particular, for the Cabibbo angle we can obtain $s_{12} = \sqrt{d/s}$. Unfortunately, the Fritzsch texture seems to be already excluded by recent CDF bound on the top mass. However, there are some other suggestions [17] which still agree to the experimental data.

The idea of radiative mass generation [18] in general aims to provide rather qoulitative explanation to the fermion mass hierarchy. Indeed, it is tempting to

think that the $1-2$ orders of magnitude hierarchy between fermion masses (see eqs. (1) and (2)) is due to loop expansion: $\varepsilon \sim (h^2/16\pi^2)$ with h being a typical Yukawa coupling of the order of 1. This could be, if due to some symmetry reasons only the heaviest 3^{rd} family gets mass at tree level, whereas the 2^{nd} becomes massive at the 1-loop level and the 1^{st} only at 2-loops. The models exhibiting this feature, were suggested in [19, 20]. However, the radiative models fail in predictivity. Moreover, it is rather difficult to obtain a quantitatively consistent picture, and also to avoide dangerous flavour changing phenomena [20, 21, 22].

The general feature of the frameworks considered above is that the mass generation starts from the heaviest third family and then propagates to the lighter ones. However, the fermion mass pattern may hint that the case is just the opposite, and the first family plays an unique role in mass generation. We stick to the observation that the GUT scale running masses of the electron, u-quark and d-quark are not strongly split, which maybe manifests the approximate symmetry limit. With this picture in mind, it is suggestive to think that the masses of the 1^{st} family are somehow related to an energy scale M_1 at which this symmetry is still good, while the masses of the 2^{nd} and 3^{rd} family are respectively related to lower scales M_2 and M_3 at which it is no longer as good. Suppose that the first family is indeed the starting point, and that the expression like eq. (1) holds for the inverse masses rather then masses, namely

$$\frac{1}{f_i} \sim \frac{\varepsilon_f^{i-1}}{m}\,, \qquad f = u, d, e \tag{6}$$

where $i = 1, 2, 3$ is a family number. Then we have $u \sim d \sim m$, $c/s \sim (\varepsilon_d/\varepsilon_u) > 1$ and $t/b \sim (\varepsilon_d/\varepsilon_u)^2 \gg 1$. In this way, the splitting between up and down quark masses in Fig. 1 is understood by means of one parameter $\varepsilon_d/\varepsilon_u > 1$. We call the above formula the *inverse hierarchy pattern*.

The above consideration suggests that the mass generation proceeds from the lightest family to heavier ones. At first sight, it is nonsense. However, we do not imply this literally. Let us take here some break and go back to neutrinos.

The $SO(10)$ extension (in fact, already the $L-R$ symmetric model) brings new particles in addition to the minimal fermion spectrum of the SM. These are right handed neutrinos. However, it is not expected that they will be seen at lower energies. After $SO(10)$ breaking down to SM no symmetry defences them to be massless, so they should acquire $O(M_G)$ Majorana mass terms and thereby decouple from the light particle spectrum. On the other hand, the $SO(10)$ invariant Yukawa couplings provide the neutrino Dirac mass terms, which in the standard picture resemble the up quark masses. The interplay of both mass structures results in the famous seesaw mechanism, which provides the naturally small majorana masses to physical neutrinos [23]. The resulting neutrino mass matrix reads as

$$\hat{m}_\nu = v^2 \Gamma \hat{M}_R^{-1} \Gamma^T \tag{7}$$

where $\hat{M}_R$ is a Majorana mass matrix of right handed neutrinos and Γ is a matrix of the "Dirac" Yukawa couplings.

One can imagine, that there are also some charged fermion states which are allowed to be superheavy by GUT symmetry. For example, already further extension of the $SO(10)$ to the E_6 model brings such heavy states. Then it is suggestive to think that the masses of ordinary quarks and leptons could appear through the analogous seesaw mixing with these heavy states. Such a possibility, named subsequently as an *universal seesaw mechanism*, was suggested in [24]. Then for the quark and lepton mass matrices we have the expression analogous to eq. (7).

It seems quite natural to assume that the Yukawa couplings all are $O(1)$ and the fermion mass hierarchy is initiated in the heavy fermion sector, while the usual light fermions are just the spectators of this phenomenon. Then the inverse power in eq. (7) is crucial: by means of the seesaw mechanism this hierarchy will be transfered to ordinary fermions in an inverted way.[3)] This can provide a firm basis to the inverse hierarchy pattern of the eq. (6). It is clear, that the heaviest ones among the heavy fermions are just the partners of the 1^{st} standard family, and its small mass splitting can be just a reflection of the symmetry limit: namely, these heaviest of heavies can be so heavy [26], in particular, heavier than the relevant GUT scale, that their mass terms obey the isotopic and quark-lepton symmetries, which are the natural subsymmetries e.g. of the $SO(10)$.

The seesaw induced inverse hierarchy pattern was explored in radiative mass generation scenario for quarks [22, 31] and also included leptons [32]. As a result, several intriguing predictions were obtained for the fermion masses and mixing angles. It is clear, however, that the use of a radiative mechanism to generate the mass hierarchy in a heavy fermion sector is in obvious contradiction with the idea of low-energy supersymmetry. Within SUSY scheme one should think of some tree level mechanism that could generate the masses of heavy fermions by means of the effective operators of successively higher dimension, thus providing a hierarchical structure to their mass matrix.

Before proceeding let us comment also, that universal seesaw can automatically solve the strong CP-problem without introducing an axion, *a lá* Nelson-Barr mechanism [27]. Such models were suggested in [28, 29] on the basis of the spontaneously broken P-parity [28] or CP-invariance [29], where the Θ-term is automatically vanishing at tree level and appears to be naturally small due to loop corrections. Alternatively, within the seesaw picture one can naturally incorporate the Peccei-Quinn type symmetries [24, 30], where the axion appears to be simultaneously a majoron.

[3)] The idea of universal seesaw mechanism was also explored in a number of papers [25]. The inverse hierarchy, however, corresponds to the spirit of the original paper [24], where it was in fact first suggested.

3. Inverse Hierarchy in SUSY SO(10) Model

We intend to built a predictive SUSY $SO(10)$ model for the fermion masses, pursuing the universal seesaw mechanism in order to obtain naturally the inverse hierarchy pattern. For this purpose one has to appeal to some symmetry properties. We suggest that there is some "family-type" symmetry (discrete or global) $\mathcal{H}$, that distinguishes the superfields involved into the game. In the following we will not specify the exact form of $\mathcal{H}$, describing only the pattern how it should work. We also wish that our model fulfills the following fundamental conditions:

A. Unification of the strong, weak and weak hypercharge gauge couplings – correct prediction for $\sin^2\theta_W$ or α_s at lower energies.

B. Natural (not fine-tuned) gauge hierarchy and doublet-triplet splitting – a couple of Higgs doublets should remain light whereas their colour triplet partners in GUT supermultiplet must be superheavy.

C. Sufficiently long-lived proton – proton lifetime should be above the recent experimental lower bound $\tau_p > 10^{32}$ yr.

D. Natural suppression of the FCNC.

Let us design such a SUSY $SO(10)\otimes\mathcal{H}$ model. We know that three families of quarks and leptons should be arranged within 16-plets 16^f_i, $i=1,2,3$. Besides them, I exploit three families of superheavy fermions $16^F_k + \overline{16}^F_k$. All these have certain transformation properties under $\mathcal{H}$ symmetry. For the following it is convenient to describe them in the terms of $SU(4)\otimes SU(2)_L\otimes SU(2)_R$ decomposition:

$$16^f_i = f_i(4,2,1) + f^c_i(\bar{4},1,2) \tag{8}$$

$$16^F_i = \mathcal{F}_i(4,2,1) + F^c_i(\bar{4},1,2)\,, \qquad \overline{16}^F_i = \mathcal{F}^c_i(\bar{4},2,1) + F_i(4,1,2) \tag{9}$$

For the electroweak symmetry breaking and quark and lepton mass generation we use a traditional 10-dimensional Higgs supermultiplet

$$10 = \phi(1,2,2) + T(6,1,1) \tag{10}$$

For the $SO(10)$ symmetry breaking we promote, as usual, a set of scalar superfields, consisting of various 54-plets, 45-plets and 126-plets, which also have different transformation properties under $\mathcal{H}$.

$$\begin{aligned} 54 &= (1,1,1) + (1,3,3) + (20,1,1) + (6,2,2) \\ 45 &= (15,1,1) + (1,3,1) + (1,1,3,) + (2,2,6) \\ 126 &= (10,1,3) + (\overline{10},3,1) + (6,1,1) + (15,2,2) \end{aligned} \tag{11}$$

We suggest that all 54-plets have the VEVs of standard configuration corresponding to the symmetry breaking channel $SO(10)\to SU(4)\otimes SU(2)_L\otimes SU(2)_R$. As for the 45-plets, we suggest that there are three types of them: 45_{BL}-type fields with VEV on their (15,1,1) fragment, breaking $SU(4)$ down to $SU(3)_c\otimes U(1)_{B-L}$; 45_R-type

fields with VEV on the (1,1,3) component, providing the breaking $SU(2)_R \to U(1)_R$; and 45_X-type fields having the VEVs on both (15,1,1) and (1,1,3) components:

$$\begin{array}{l} \langle 54 \rangle = I \otimes \mathrm{diag}(1,1,1,-3/2,-3/2) \cdot V_G \\ \langle 45_{BL} \rangle = \sigma \otimes \mathrm{diag}(1,1,1,0,0) \cdot V_{BL} \\ \langle 45_R \rangle = \sigma \otimes \mathrm{diag}(0,0,0,1,1) \cdot V_R \quad ; \\ \langle 45_X \rangle = \sigma \otimes \mathrm{diag}(1,1,1,x,x) \cdot V_X \end{array} \qquad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{12}$$

Finally, the 126-plet with VEV v_R across the (10,1,3) direction completes the $SO(10)$ breaking down to $G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. Motivated by the coupling crossing phenomenon in MSSM [2], we suggest that $SO(10) \otimes \mathcal{H}$ breaks down to G_{SM} at once, by VEVs $V_G, V_X, V_{BL}, v_R \sim M_G$. Below this scale the theory is just MSSM, with three fermion families (f_i) and one light couple of the Higgs doublets (ϕ). Many questions of the series A - D are immediately respected in this way: $\sin^2\theta_W(\mu)$ and $\alpha_s(\mu)$ are correctly related at $\mu = M_Z$; the FCNC are naturally suppressed provided that SUSY breaking sector has simple (e.g. universal) structure; large unification scale ($M_G \simeq 10^{16}$ GeV) saves the proton from the unacceptably fast decay mediated by X, Y gauge bosons ($d = 6$ operators). Let us comment also, that the SUSY $SO(10)$ theory has automatic matter parity, under which the spinorial representations change the sign while the vectorial ones stay invariant.[4)] Provided that non of the 16-plets have the VEV, this implies an automatic R-parity conservation for the resulting MSSM. It is well-known, that proton decaying $d = 4$ operators mediated by squarks are vanishing in this case.

In order to establish the seesaw regime at once, we assume that $\mathcal{H}$ symmetry does not allow $16^f 16^f 10$ couplings but only the following terms in the Yukawa superpotential:

$$\Gamma_{ik} 10\, 16^f_i 16^F_k + G_{ik} 45_R\, 16^f_i \overline{16}^F_k \tag{13}$$

These couplings generate the mass terms for ordinary quarks and leptons (f-fermions) by means of seesaw mixing with the F-fermion states afterthat the latter become superheavy. The whole 9×9 mass matrices have the form

$$M^f_{tot} \; = \; \begin{array}{c} \\ f \\ F \\ \mathcal{F} \end{array} \!\!\! \begin{array}{c} \begin{array}{ccc} \;\; f^c \;\; & \;\; F^c \;\; & \;\; \mathcal{F}^c \end{array} \\ \begin{pmatrix} 0 & \hat{M}_{fL} & 0 \\ \hat{M}_{fR} & \hat{M}_F & 0 \\ \hat{M}^\dagger_{fL} & 0 & \hat{M}_{\mathcal{F}} \end{pmatrix} \end{array} \tag{14}$$

Where $\hat{M}_{fL} = \Gamma \langle \phi \rangle$ and $\hat{M}_{fR} = G^T \langle 45_R \rangle$. We do not specify further the form of the Yukawa couplings. We only suggest that they all are $O(1)$, as well as the gauge coupling constants. Γ and G are some general complex and non-degenerated matrices. Without loss of generality, by suitable simultaneous redefinition of the

[4)] In fact, this gives a natural ground to refer the spinorial representations as fermionic superfields, and the vectorial ones as Higgs superfields.

basis of f fermions of all types ($f = u, d, e, \nu$), we always can bring them to a skew-diagonal form:

$$\Gamma_{ik},\, G_{ik} = 0\,, \qquad \text{if} \quad i < k. \tag{15}$$

The role of the Higgs 10-plet is crucial. We require that its $\phi(1,2,2)$ component, which consists of the Higgs doublets, remains massless in the SUSY limit. On the other hand, the $T(6,1,1)$ fragment, containing colour triplets, should acquire the mass of the order of M_G: otherwise it would cause unacceptably fast proton decay and would affect the unification of the gauge couplings. In order to resolve this famous problem of the doublet-triplet splitting without fine tuning of the superpotential parameters, one can address to the Dimopoulos-Wilczek mechanism, utilizing the Higgs 45_{BL}-plet [33]. The VEV of ϕ arises after the SUSY breaking and breaks the $SU(2)_L \otimes U(1)_Y$ symmetry:

$$\langle\phi\rangle = \begin{pmatrix} v_2 & 0 \\ 0 & v_1 \end{pmatrix}, \qquad (v_1^2 + v_2^2)^{1/2} = v = 175\,\mathrm{GeV} \tag{16}$$

This implies that the $(1,2)$-blocks of the matrix M^f_{tot} are essentially the same: $\hat{M}_{fL} = \Gamma v \sin\beta$ for the up-type fermions ($f = u, \nu$) and $\hat{M}_{fL} = \Gamma v \cos\beta$ for the down-type ones ($f = d, e$), where $\tan\beta = v_2/v_1$ is the famous up-down VEV ratio in MSSM.

Equally important is the choice of the VEV $\langle 45_R\rangle$ towards the $(1,1,3)$ direction. It tells that the $(2,1)$-block of M^f_{tot} differs *only* by the sign for the up-type and down-type fermions: $\hat{M}_{fR} = +G^T V_R$ for $f = u, \nu$ and $\hat{M}_{fR} = -G^T V_R$ for $f = d, e$. On the other hand, it implies that the $(1,3)$-block is vanishing. Therefore, only the $SU(2)_L$-singlet F-type fragments of eq. (9) are important for the seesaw mass generation, whereas the $\mathcal{F}$-type ones are irrelevant. As it was shown in [34], this feature is decisive for the natural suppression of the dangerous $d = 5$ operators for the proton decay.[5)]

Therefore, we have all the key ingredients for the inverse hierarchy ansatz. What remains is to obtain the needed hierarchical pattern for the heavy mass matrices $\hat{M}_F$. Let us assume that the $\mathcal{H}$ symmetry allows the bare mass term ($M \gg M_G$) for the 1^{st} heavy family F_1 and the mass of the 2^{nd} one (F_2) is generated via 45_X:

$$M 16^F_1 \overline{16}^F_1 \; + \; g 45_X 16^F_2 \overline{16}^F_2\,, \tag{17}$$

while the 3^{rd} family becomes massive through the effective operator $(45_X^2/M) 16^F_3 \overline{16}^F_3$. In this case the fermion mass hierarchy will be explained due to small parameter $\varepsilon \sim V_G/M$. However, it is not enough restrictive to use this later operator without

[5)] Indeed, the f and $\mathcal{F}$ states are unmixed, so the colour triplets in $T(6,1,1)$ can cause transitions of f's only into the superheavy $\mathcal{F}$'s. Therefore, the baryon number violating $d = 5$ operators $[ffff]_F$ (so called $LLLL$ type operators), which bring the dominant contribution to the proton decay after dressing by the $\tilde{W}$-bosinos, are automatically vanishing. As for the $RRRR$ type operators $[f^c f^c f^c f^c]_F$, they clearly appear due to the $f^c - F^c$ mixing. But they are known to be much more safe for the proton (see e.g. [35] and refs. therein).

defining to which of the possible $SO(10)$ channels it acts: $45 \times 45 \to 1 + 45 + 210$. In order to be less vague, let us introduce the additional couple $16^F_0 + \overline{16}^F_0$ with barr mass $M' \sim M$ and Yukawa couplings $g' 16^F_3 \overline{16}^F_0 + g'' 16^F_0 \overline{16}^F_3$.[6] Then the mass terms of the F_3 states appear at the decoupling of the heavier F_0 states, i.e. after the diagonalization of the mass matrix

$$\begin{array}{cc} & \begin{array}{cc} F_3^c & F_0^c \end{array} \\ \begin{array}{c} F_3 \\ F_0 \end{array} & \left(\begin{array}{cc} 0 & g'\langle 45_X \rangle \\ g''\langle 45_X \rangle & M' \end{array} \right) \end{array} \tag{18}$$

As a consequence, we obtain the mass matrices $\hat{M}_F$ of the desired form:

$$\hat{M}_F = M(\hat{P}_1 + \varepsilon_f \hat{P}_2 + \varepsilon_f^2 \hat{P}_3)\,, \qquad \begin{array}{c} \hat{P}_1 = \mathrm{diag}(1,0,0) \\ \hat{P}_2 = \mathrm{diag}(0,1,0) \\ \hat{P}_3 = \mathrm{diag}(0,0,C) \end{array} \tag{19}$$

where $C \sim M/M' \sim 1$, since all Yukawa couplings are assumed to be $O(1)$. What is new, is that the complex expansion parameters ε_f ($f = u, d, e, \nu$) *are not* independent anymore, but are related due to the VEV pattern (12) of the 45_X:

$$\begin{array}{ll} \varepsilon_d = \varepsilon_1 + \varepsilon_2\,, & \varepsilon_e = -3\varepsilon_1 + \varepsilon_2 \\ \varepsilon_u = \varepsilon_1 - \varepsilon_2\,, & \varepsilon_\nu = -3\varepsilon_1 - \varepsilon_2 \end{array} \tag{20}$$

from where follows

$$\varepsilon_e = -\varepsilon_d - 2\varepsilon_u\,, \qquad \varepsilon_\nu = 2\varepsilon_e + 3\varepsilon_u \tag{21}$$

Assuming $\hat{M}_F \gg GV_R$, the seesaw block-diagonalization of eq. (14) results in following mass matrices for the ordinary quarks and leptons:

$$\hat{m}_f = \zeta_f v V_R \Gamma \hat{M}_F^{-1} G^T \tag{22}$$

where $\zeta_f = \sin\beta$ for $f = u, \nu$ and $\zeta_f = -\cos\beta$ for $f = d, e$. In this way the inverse proportionality of eq. (6) is realized. The seesaw limit $M_F \gg V_R$ is certainly very good for all light states apart from t-quark, since their masses must be much smaller than v. However, since $m_t = O(v)$, we expect the mass of its F-partner M_T to be of the order of V_R (remember that the Yukawa couplings are considered to be $O(1)$).[7]

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[6] For the simplicity we assume that F_0 has no couplings with the 16^f's, though it is easy to see that such couplings would not affect significantly our results. The possible contributions of the 10-plet couplings to heavy states (9) are also negligible.

[7] Decoupling of the heavy states F occurs at the scale V_R: below this scale the effective theory is the MSSM, and $V_R \Gamma \hat{M}_F^{-1} G^T = \hat{m}_f / v\zeta_f$ are in fact the MSSM Yukawa couplings. Then the ratio V_R/M is given by $m_u/v \sim 10^{-5}$. Taking into account that $M/M_G \sim \varepsilon_d \sim 30$, this implies $V_R \sim 10^{13}$ GeV, i.e. some three order of magnitude below the GUT scale M_G. This is not a

Thus, to evaluate m_t, we need the mass matrix without the restriction $\hat{M}_F \gg V_R$. This is given by

$$\hat{m}_f \hat{m}_f^\dagger = (\zeta_f v)^2 \Gamma \left[1 + \hat{M}_F^\dagger (G^T G^* V_R^2)^{-1} \hat{M}_F\right]^{-1} \Gamma^\dagger . \tag{23}$$

Notice that, when $V_R \gg \hat{M}_F$, this equation gives the obvious result $\hat{m}_f = \Gamma \zeta_f v$. On the other hand, when $V_R \ll \hat{M}_F$, it reduces to the seesaw formula (22).

It is useful to first study $\hat{m}_f$ in the seesaw limit (22). The exact formula (23) will only be relevant to evaluate m_t. Once again, the inverse matrices are easier to analyse. From the eqs. (19) and (22) we have

$$\hat{m}_f^{-1} = \frac{M}{v\zeta_f V_R}(G^T)^{-1}(\hat{P}_1 + \varepsilon_f \hat{P}_2 + \varepsilon_f^2 \hat{P}_3)\Gamma^{-1} = \frac{1}{m\zeta_f}(\hat{Q}_1 + \varepsilon_f \hat{Q}_2 + \varepsilon_f^2 \hat{Q}_3) , \tag{24}$$

where $\hat{Q}_n \propto (G^T)^{-1}\hat{P}_n \Gamma^{-1}$ are still rank-1 matrices, but not orthogonal anymore. We can also choose a basis of eq. (15) and use a relation $(G^T)^{-1}\hat{P}_1\Gamma^{-1} = (G_{11}\Gamma_{11})^{-1}\hat{P}_1$, to define $\hat{Q}_1 = \hat{P}_1$ and $m = \Gamma_{11} G_{11} v V_R / M$. In other words, without loss of generality we can take

$$\hat{Q}_1 = (1,0,0)^T \bullet (1,0,0), \quad \hat{Q}_2 = (a,b,0)^T \bullet (a',b',0), \quad \hat{Q}_3 = (x,y,z)^T \bullet (x',y',z') \tag{25}$$

so that the inverse mass matrices at the leading order are the following:

$$\hat{m}_f^{-1} = \frac{1}{m\zeta_f} \begin{pmatrix} 1 + aa'\varepsilon_f & ab'\varepsilon_f & xz'\varepsilon_f^2 \\ ba'\varepsilon_f & bb'\varepsilon_f & yz'\varepsilon_f^2 \\ zx'\varepsilon_f^2 & zy'\varepsilon_f^2 & zz'\varepsilon_f^2 \end{pmatrix} \tag{26}$$

Here have we neglected $O(\varepsilon)$ order corrections in every element except the 11-one. In order to split fermion masses within the first family and accomodate large ($\sim \sqrt{\varepsilon_d}$) Cabibbo angle, the matrix (26) must be diagonalized considering that $aa'\varepsilon_{d,e} \sim 1$.[8)]

big problem neither for gauge coupling unification nor for other issues, provided that the VEV of the 126-plet v_R is of the order of M_G. Nevertheless, one may does not consider such a small V_R as enough appealing. In this case we can suggest that the $(2,1)$-block $\hat{M}_{fR}$ of the "big" mass matrix (14) appears due to the effective operators $(45_R^2/M) 16_f \overline{16}_F$ rather than the direct Yukawa couplings of the eq. (13). These operators can be built in the same manner as we did for the third heavy family F_3. Then $\hat{M}_{fR} \sim 10^{13}$ GeV can occur naturally for $V_R \sim M_G$, and non of our results will change.

8) One may wonder how to achieve $\varepsilon_d aa' \simeq 1$, if the Yukawa couplings are assumed to be $O(1)$ and ε is a small parameter: $\varepsilon_d \sim 1/20 - 1/30$ (see eq. (1)). However, here we still see the advantage of seesaw mechanism: a and a' are not the coupling constants but rather their ratios, due to the "sandwiching" between Γ and G in eq. (22). Thus, it should not come as a surprise if $aa' \sim 20-30$ due to some spread in the Yukawa coupling constants (for example, if both $a = \Gamma_{21}/\Gamma_{11}$ and $a' = G_{21}/G_{11}$ are $\sim 4-5$), while the Yukawa constants themselves are small enough to fulfill the triviality bound $G_Y^2/4\pi < 1$. On the other hand, the pattern of the fermion masses and mixing suggests that such a "coherent" enhancement does not happen for other entries in the matrix (22), so that the corresponding $O(\varepsilon)$ corrections are negligible.

Thus, for the fermion mass eigenvalues at the GUT scale we have

$$\begin{aligned}
\frac{m\sin\beta}{u} &= |1+\varepsilon_u aa'|\,, & \frac{m\sin\beta}{c} &= \frac{|\varepsilon_u bb'|}{|1+\varepsilon_u aa'|}\,, & \frac{m\sin\beta}{t} &= |\varepsilon_u^2 zz'| \\
\frac{m\cos\beta}{d} &= |1+\varepsilon_d aa'|\,, & \frac{m\cos\beta}{s} &= \frac{|\varepsilon_d bb'|}{|1+\varepsilon_d aa'|}\,, & \frac{m\cos\beta}{b} &= |\varepsilon_d^2 zz'| \\
\frac{m\cos\beta}{e} &= |1+\varepsilon_e aa'|\,, & \frac{m\cos\beta}{\mu} &= \frac{|\varepsilon_e bb'|}{|1+\varepsilon_e aa'|}\,, & \frac{m\cos\beta}{\tau} &= |\varepsilon_e^2 zz'|
\end{aligned} \tag{27}$$

From the two first rows of eqs. (27) we have

$$\left|\frac{\varepsilon_u}{\varepsilon_d}\right| = \frac{ds}{uc}\tan^2\beta\,, \qquad \left|\frac{\varepsilon_u}{\varepsilon_d}\right|^2 = \frac{b}{t}\tan\beta \quad \Longrightarrow \quad \frac{t}{b} = \left(\frac{uc}{ds}\right)^2 \tan^{-3}\beta \tag{28}$$

This expression for the top mass is valid in the seesaw limit of eq. (22). However, due to seesaw corrections, it actually gives only an upper bound. Indeed, by using the correct mass matrix of eq. (23), the eq. (28) is reduced to

$$t = \frac{bR}{\sqrt{1+(bR/\Gamma_{33}v\sin\beta)^2}} < bR\,; \qquad R = (uc/ds)^2\tan^{-3}\beta \tag{29}$$

where for the perturbativity one can assume $\Gamma_{33} \leq 2$. This equation is valid at the GUT scale and to discuss its implications, the running of masses needs to be considered. In doing so, it appears to be rather restrictive. In particular, by taking $m_u/m_d \leq 0.7$ and $m_c/m_s \leq 12$ as upper bounds and bearing in mind that $\tan\beta \geq 1$, we get

$$R \leq R_{\mathsf{max}} \simeq 70 \tag{30}$$

which sets an upper bound on the top physical mass m_t of about 150 GeV. On the other hand, by taking the recent CDF bound $m_t > 109$ GeV we have

$$R \geq R_{\mathsf{min}} = 36 \tag{31}$$

which translates into the strong upper bound $\tan\beta < 1.1$ for the same values of m_u/m_d and m_c/m_s. Alternatively, by assuming $\tan\beta = 1$, we have a lower bound $uc/ds > 6$. Then, by using $m_c/m_s < 12$, we get $m_u/m_d > 0.5$.

From the second two rows of eqs. (27) we can derive the mass formulae

$$\sqrt{\frac{b}{\tau}} = \frac{ds}{e\mu} = \left|\frac{\varepsilon_e}{\varepsilon_d}\right| = \left|1 + 2\,\frac{\varepsilon_u}{\varepsilon_d}\right| \tag{32}$$

When I get to the bottom I go back to the top [36]: the eq. (28) shows that the $\varepsilon_u/\varepsilon_d$ ratio is small: $|\varepsilon_u/\varepsilon_d| = 0.12 \div 0.16$. Then the eq. (32) approximately gives the GUT relationships between the down quark and lepton masses:

$$b = \tau\,, \qquad ds = e\mu \tag{33}$$

where $O(\varepsilon_u/\varepsilon_d)$ corrections bring about 30 % uncertainty. Running down these relations from the GUT scale we get appealing values for the down quark masses: $m_b = 4-5$ GeV, $m_s = 100-150$ MeV and $m_d = 5-7$ MeV, where for deriving the light quark masses we have used the current algebra relation $m_s/m_d = 20$. This later relation, however, cannot be derived from our consideration itself. Moreover, it is not clear whether it is consistent in our scheme: at first sight the relation $|\varepsilon_d| \approx |\varepsilon_e|$ implies that $d/s \approx e/\mu \sim |\varepsilon_e|$, whereas the experimental values $s/d \approx 20$ and $\mu/e \approx 200$ differ by the order of magnitude.

The answer is "Yes"! Indeed, by taking into account that $\tan\beta \approx 1$, we get:

$$u/d = |1+\varepsilon_d aa'|\,, \qquad u/e = |1+\varepsilon_e aa'| \tag{34}$$

and

$$\frac{(s/d)}{(\mu/e)} = \frac{|1+\varepsilon_d aa'|^2}{|1+\varepsilon_e aa'|^2} = \left(\frac{e}{d}\right)^2 = \left(\frac{s}{\mu}\right)^2 \tag{35}$$

where we have neglected $O(\varepsilon_u/\varepsilon_d)$ corrections. Therefore, in order to split the electron and d-quark masses from the u-quark mass respectively by factors of about $\frac{1}{2}$ and 2 (see Fig. 1), we have to assume that $|\varepsilon_e aa'| \approx 1$. The relation $\varepsilon_d \approx -\varepsilon_e$ is crucial, since it splits d and e to different sides from $u \approx m$ by about a factor 2. Then, according to eq. (35), the order of magnitude difference between μ/e and s/d follows automatically. In this way, owing to the numerical coincidence $(e/d)^2 \sim \varepsilon_u\varepsilon_d$, we reproduce the mixed behaviour of leptons (see eq. (2)). This is not, however, the end of the story. By assuming that $|\varepsilon_e aa'| \leq 1$ (which, as we show below, can be derived by considering the quark mixing), the eqs. (34) and (35) imply

$$\frac{m_u}{m_d} = |1+\varepsilon_e aa'| \left(\frac{m_e m_s}{m_\mu m_d}\right)^{1/2} \leq 0.65 \tag{36}$$

where the $O(\varepsilon_u/\varepsilon_d)$ corrections can cause about 20 % uncertainty in this estimate.

Let us discuss now the pattern of the weak mixing. It is easy to see that the quark mixing arises dominantly due to diagonalization of the down quark mass matrix $\hat{m}_d$. The up quark matrix $\hat{m}_u$ is much more "stretched" and essentially close to its diagonal form, so that it brings only $O(\varepsilon_u/\varepsilon_d)$ corrections to the CKM mixing angles. Let us denote by $\theta_{12}^{L,R}$, $\theta_{23}^{L,R}$ and $\theta_{23}^{L,R}$ the angular parameters of the unitary matrices $V_{L,R}$ diagonalizing $\hat{m}_d$. Then from the eqs. (26) and (27) we see that

$$s_{12}^L = \frac{|\varepsilon_d ab'|}{|1+\varepsilon_d aa'|}\,, \quad s_{12}^R = \frac{|\varepsilon_d a'b|}{|1+\varepsilon_d aa'|} \quad \Longrightarrow \quad s_{12}^L s_{12}^R = \frac{d}{s}\,|\varepsilon_d aa'| \tag{37}$$

where "$s^{L,R}$" stand for $\sin\theta^{L,R}$. Then, by assuming that the right-handed current "Cabibbo" angle s_{12}^R is not anomalously large (not larger than the ordinary Cabibbo angle $s_{12}^L = s_{12} \approx \sqrt{d/s}$, which in itself is already much larger than it was expected naively ($\sim \varepsilon_d$) from the eq. (37)), we obtain that $|\varepsilon_d aa'| \leq 1$. On the other hand, we

know that $|\varepsilon_d aa'|$ should be rather close to 1, in order to achieve a sufficient $d-u-e$ splitting. By assuming that $s_{12}^R \sim s_{12}$, the eq. (37) translates into

$$s_{12} \approx \sqrt{\frac{d}{s}\left|1-\frac{u}{d}e^{i\delta_d}\right|}, \qquad \delta_d = \arg|1+\varepsilon_d aa'| \tag{38}$$

Thus, the Cabibbo angle has to be in the right range: $s_{12} \sim \varepsilon_d^{1/2}$. Obviously, the values of other mixing angles also fit parametrically the pattern of the eq. (4): $s_{23} \sim \varepsilon_d$ and $s_{13} \sim \varepsilon_d^2$. Without exploiting the concrete structures of the Yukawa coupling matrices Γ and G it is not possible to make exact predictions for the CKM matrix parameters. It is natural to expect, however, that the $\mathcal{H}$ symmetry will constrain somehow the form of Γ and G, and thereby will enhance the predictivity. Let us assume, for example, that Γ has the Fritzsch form (5), involving 5 parameters. Then, rotating the fields to the skew-diagonal basis of eq. (15), we see that 6 complex entries of Γ are not independent anymore, but are related through $\Gamma_{31} = -\Gamma_{21}\Gamma_{22}\Gamma_{32}^{-1}$. The same can occur for the matrix G. Then we immediately receive an appealing relation between the CKM mixing angles

$$\frac{s_{13}}{s_{23}} = \frac{u}{d}\, s_{12} = 0.11 \div 0.15 \tag{39}$$

However, there is a subtlety which we have avoided to discuss untill now. Obviously, one 10-plet is not sufficient to obtain the realistic mass matrices. The reason is that the symmetry $\mathcal{H}$ should transform the fermionic superfields in different way (in order to assure the form of the heavy mass matrices (19) by symmetry reasons). Therefore, 10-plet is allowed to have at most 3 non-zero Yukawa couplings in the eq. (13). The same is true for the 45_R. Obviously, three non-zero entries in the matrices Γ and G are not enough - they will appear to be diagonal or degenerated. In order to supply in (14) the off-diagonal entries $\hat{M}_{fL}$ and $\hat{M}_{fR}$ of the non-trivial form, we need at least two 10's and two 45_R's: $10_{1,2}$ and $45^R_{1,2}$, with different transformation properties under $\mathcal{H}$, and all with non-zero VEVs. The Yukawa superpotential is

$$W_Y = \Gamma^A_{ik} 10_A\, 16^f_i 16^F_k + G^B_{ik} 45^R_B\, 16^f_i \overline{16}^F_k \tag{40}$$

Provided that both 45_R-plets have non-zero VEVs on the $(1,1,3)$ component, which also break $\mathcal{H}$ symmetry, we can change the basis and single out one linear combination $45_R \propto V_{R1}45^R_1 + V_{R2}45^R_2$, which takes all the effective VEV $V_R = (V_{R1}^2 + V_{R2}^2)^{1/2}$. The other combination has vanishing VEV.

For the 10-plets the situation is more specific. In order not to affect the gauge coupling crossing, only one combination of the two ϕ_1 and ϕ_2, that are (1,2,2) components of 10_1 and 10_2, should remain massless (in the SUSY limit), whereas other has to acquire $O(M_G)$ mass. On the other hand, the (6,1,1) fragments T_1 and T_2 both should be superheavy. Also, we do not want to pay *fine tuning* for this doublet-triplet splitting. To achieve this, we suggest to modify the Dimopoulos-Wilczek mechanism [33] in the following manner. Let us introduce yet another

10-plet, 10_0, not necessarily coupled to fermions, and assume that the $\mathcal{H}$ symmetry is designed so that allows only the following couplings for 10's:

$$\lambda 10_1 10_2 45_{BL} + \lambda_1 10_1 10_0 45_1^R + \lambda_2 10_2 10_0 45_2^R + \lambda_0 10_0 10_0 54 \tag{41}$$

Therefore, after substituting the relevant VEVs, mass matrices of the ϕ and T components of the 10's are

$$M_{\phi,T} \; = \; \begin{array}{c} \\ 10_1 \\ 10_2 \\ 10_0 \end{array} \begin{array}{c} \begin{array}{ccc} \quad 10_1 \quad & \quad 10_2 \quad & \quad 10_0 \quad \end{array} \\ \left(\begin{array}{ccc} 0 & \lambda\langle 45_{BL}\rangle & \lambda_1\langle 45_1^R\rangle \\ -\lambda\langle 45_{BL}\rangle & 0 & \lambda_2\langle 45_2^R\rangle \\ -\lambda_1\langle 45_1^R\rangle & -\lambda_2\langle 45_2^R\rangle & \lambda_0\langle 54\rangle \end{array} \right) \end{array} \tag{42}$$

Considering T-components, we see that all three eigenstates are superheavy. For the $(1,2,2)$ components we must substitute $\langle 45_{BL}\rangle \to 0$ and $\langle 45^R_{1,2}\rangle \to V_{R1,2}$. Then one linear combination $\phi \propto \lambda_2 V_{R2}\phi_1 - \lambda_1 V_{R1}\phi_2$ is massless, whereas the orthogonal combination is superheavy. The VEV of ϕ (16), which arises after the SUSY breaking, is shared between the $\mathcal{H}$ symmetry eigenstates 10_1 and 10_2.

The above consideration demonstrates how one could supply the realistic structure for the fermion mass matrices. The form of the Matrices Γ_A and G_B, $A, B = 1, 2, ...$, are constrained by the $\mathcal{H}$ symmetry: no more than three non-zero entries are allowed for each of them. However, for the linear combinations ϕ and 45_R the eq. (40) is effectively reduced to (13), where the Yukawa matrices Γ and G can have some non-trivial (e.g. Fritzsch) form.

Accidentally, the structure of the eq. (42) outwardly recembles the one suggested by Babu and Barr [37] for the *strong* suppression of the Higgsino mediated $d = 5$ operators for the proton decay. In our case, however, the *strong* suppression *a la'* Babu and Barr does not occur, since both 10_1 and 10_2 are coupled to fermions. Nevertheless, some *weak* suppression can be due to mixing of different T-states from the $10_{0,1,2}$. On the other hand, our seesaw pattern (14) in itself leads to the complete suppression of the dominant ($LLLL$-type) $d = 5$ operators, and only much weaker $RRRR$-type ones remain to be effective [34]. All this leaves us with the chance to observe the proton decay at the level of present experimental bound. It is worth to remark also, that in our scheme we can evaluate the branching ratios of the different decay modes, since we are able to calculate all mixing angles, including the ones for the charged leptons[9)] This can be rather important for the testing of our *inverse hierarchy* scheme, if the proton decay will be observed in the future.

Let us discuss now the neutrino mass and mixing pattern. Clearly, the eq. (22) is valid also for the neutrino Dirac mass matrix, which has the form of the eq. (26) with $\varepsilon_\nu = 2\varepsilon_e + 3\varepsilon_u$. Then from the equations analogous to (27) we obtain for the

[9)] In fact, the existing calculations of the proton decay modes (see e.g. [35]) cannot be satisfactory, since they are performed within the framework of the minimal SUSY $SU(5)$ model with obviously wrong mass relations $d = e$ and $s = \mu$.

neutrino Dirac mass eigenvalues at the GUT scale:

$$\nu_1^D \approx \frac{2}{3}\, e\,, \qquad \nu_2^D \approx \frac{3}{4}\, \mu\,, \qquad \nu_3^D \approx \frac{1}{4}\, \tau \tag{43}$$

These are drastically different from what is traditionally expected from the $SO(10)$ model: for example, our ν_3^D is about $200 - 300$ times less than in standard $SO(10)$ ($\nu_3^D = t$). We can also evaluate the "mixing" angles, which diagonalize the matrices $\hat{m}_e$ and $\hat{m}_\nu^D$, in the terms of the CKM angles: $s_{12}^e \approx (3/4)s_{12}^D \approx (1/3)s_{12} = 0.07$, etc. However, all these do not transform into sharp predictions for the neutrino mass and mixing pattern, since we do not know yet the Majorana mass matrix of the right handed neutrinos.

For this purpose we have the Higgs 126-plet with VEV $v_R \sim M_G$. However, there are too many different possibilities to introduce its Yukawa couplings, all with different implications for the neutrino mass end mixing pattern. For the demonstration, we consider here one of the simplest possibilities. Let us suggest that the 126-plet interacts only with the 16^F-plets: $\Lambda_{ij} 126\, 16_i^F 16_j^F$, where Λ is a coupling constant matrix with $O(1)$ elements. Then for the Majorana mass matrix of the light neutrinos we immediatelly get

$$\hat{m}_\nu^M = \frac{v^2}{2v_R} \Gamma \Lambda^{-1} \Gamma^T \tag{44}$$

This, in general, implies that all neutrinos have masses of the order of $10^{-2} - 10^{-3}$ eV, and their mixing angles are large, which favours the adiabatic MSW solution to solar neutrino problem. More precise information can be obtained by constraining the form of the matrices Γ and Λ due to $\mathcal{H}$ symmetry properties.

4. Discussion.

Let us try to give some more philosophical shape to our considerations. One could imagine that our SUSY $SO(10) \otimes \mathcal{H}$ theory is what remains from the superstring after compactification. Obviously, such a theory should be realized at some higher Kac-Moody level, since we utilize the higher dimensional representations of $SO(10)$. In particular, $k \geq 5$, if we use the 126-dimensional representation for the symmetry breaking and neutrino mass generation purposes [38]. The fermionic sector includes 5 zero modes of 16-plets: $16_{1,2,3}^f$ and $16_{2,3}^F$, and 2 zero modes of $\overline{16}$-plets: $\overline{16}_{2,3}^F$. We have also included in game some non-zero modes like $16_{0,1}^F + \overline{16}_{0,1}^F$, with masses of the order of the compactification scale $M \sim$ few times 10^{17} GeV. Taking seriously the coupling crossing phenomenon in MSSM, we suggest that the breaking of $SO(10) \otimes \mathcal{H}$ symmetry down to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ occurs at one step, at the scale $M_G \sim 10^{16}$ GeV. All what remains below is just MSSM with three quark-leptonic families that are fragments of the $16_{1,2,3}^f$, and one couple of Higgs doublets, originated from certain effective combination of the $\phi_A(1,2,2)$ components of the various 10_A. In

order to render the couple of Higgs doublets massless in the exact SUSY limit, we have used an intriguing modification of the Dimopoulos-Wilczek mechanism.

We assumed that the generation of fermion masses occurs due to universal seesaw mechanism. Once again we would like to stress that in seesaw picture the ordinary light fermions f are just the spectators of the phenomena that determine the flavour structure. This structure arises in a sector of the superheavy F fermions and is transferred to the light ones at their decoupling. Namely, the heaviest F family F_1 is unsplit since it has $SO(10) \otimes \mathcal{H}$ invariant mass of the order of M. The lighter ones F_2 and F_3 get the masses of the order of M_G and M_G^2/M respectively, due to effective operators involving the Higgs 45_X with successively increasing power, and are thereby split. As a result, the f's mass matrices, given by a seesaw mixing with the F's, have the *inverse hierarchy* form.

These mass matrices, which in eq. (24) are displayed as $\hat{m}_f^{-1}$ for the convenience reasons, reproduce the fermion spectrum (see Fig. 1) and mixing pattern in a very economical way.[10)] They differ only due to different, in general *complex* expansion parameters ε_f, $f = u, d, e, \nu$ (including the neutrino Dirac mass matrix), where $\varepsilon_f \sim M_G/M \sim 10^{-1} - 10^{-2}$. These parameters are related through the $SO(10)$ symmetry properties (see eq. (21), so only two of them, say ε_d and ε_u are independent. Due to common mass factor m, the first family plays a role of a *mass unification point*, and the $e - u - d$ mass splitting is understood by the same mechanism that enhances the Cabibbo angle up to the $O(\sqrt{\varepsilon_d})$ value. Other mixing angles naturally are in the proper range (see eq. (4)). We have obtained a number of interesting mass formulas, from which it follows that $m_t = 100 - 150$ GeV, $m_b = 4 - 5$ GeV, $m_s = 100 - 150$ Mev and $m_u/m_d = 0.5 - 0.7$. We did not utilize any particular supersymmetry breaking mechanism, therefore we do not have some certain perdictions for the parameters of MSSM. However, independently on the concrete mechanism, we have rather interesting prediction tan$\beta \approx 1$, which can be immediately tested on the accelerators of the next generation [39].

In fact, we did not suggest any concrete example of our misterious symmetry $\mathcal{H}$, that should support the inverse hierarchy pattern of fermion mass matrices, the modified Dimopoulos-Wilczek ansatz for natural doublet-triplet splitting and, in the end [40], the needed VEV pattern. In principle, $\mathcal{H}$ can contain some set of discrete or abelian (Peccei-Quinn type) symmetries. Alternatively, one can try to utilize global or discrete R-symmetries. I am convinced that to find the working example of a $\mathcal{H}$ symmetry is rather a cumbersome but available task for the smart model-builder.

We find it amusing that the idea of inverse hierarchy, implemented in SUSY $SO(10)$ theory in a natural way, can explain the key features of the fermion mass

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[10)] Clearly, both the *decoupling* and *scaling* hypothesis of the Sect. 1 are naturally fulfilled in this way. In the limit when $\varepsilon_u = \varepsilon_d$ we have the *scaling*: $u : c : t = d : s : b$ and all CKM angles are vanishing. The *decoupling* can be seen in the following way: by putting $\varepsilon_{u,d}^2$ to zero (as parametrically smaller values compared to $\varepsilon_{u,d}$), assuming also that the third family masses t and b are fixed, we see that $u, d \to 0$ and at the same time $s_{12}, s_{13} \to 0$. At the next step, bu putting $\varepsilon_{u,d}$ to zero, we see that $c, s \to 0$ and also $s_{23} \to 0$.

spectrum and weak mixing. Notice, that in contrast to all known predictive frameworks for fermion masses (see e.g. [16, 17]), we did not exploit any particular texture - a horizontal structure that suggests an existence of certain "zeros" in mass matrices. Moreover, it is clear that in our mass matrices there can be no "zeros" - this immediately would bring us to wrong predictivity. However, a *clever* horizontal structure would enhance a predictive power of our approach. In paricular, one can expect that the $\mathcal{H}$ symmetry will constrain also the Yukawa coupling matrices Γ and G so that they will have certain pattern with certain "zero" elements. These "zeros" will not be seen directly in the quark and lepton mass matrices of the eq. (22), but they will manifest themselves through certain relations between parameters of the eq. (25) which we have treated before as independent. The example of the succesfull relation (39) obtained by suggesting the Fritzsch texture for Γ and G demonstrates that may be "zeros" are not placed directly in the mass matrices, where they are generally looked while "stitching the Yukawa quilt".

"With a little help from my friends..."

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